55280

A

**2017**
**MCM/ICM**
**Summary Sheet**

## Summary

**Keywords**

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# 1 Introduction

## 1.1 Problem Background

$9500m^3/s$ $180.6km^3$

## 1.2 Our Work

## 2 Assumptions

## 3 Nomenclature

## 4 Statement of our Model

| | fi |
|---|---|
| $i$ | $i$ |
| $X$ | $i$ |
| $Series(X)$ | |
| $Supply(X)$ | $i$ |
| $Risk(X)$ | $i$ |
| $P$ | |
| $Q(X;t)$ | fl $t$ $i$ |
| $k(X)$ | $i$ |
| $N$ | |
| $n$ | |
| $M$ | |
| $L$ | |
| $d$ | |
| $\alpha$ | |
| $h(X)$ | $i$ |

## 4.1 Behavior of Water Flow

fl fl fl

$$v = \frac{k}{n} R^{2/3} S^{1/2} \quad ( )$$

- $v$
- $n$ fi
- $R$ $(L. f, m)$.
- $S$ $(L/L)$.
- $k$

fi fl $S$ $R$.

## 4.2 Analysis of A Single Dam

**storage capacity**

$$CA(X) = \sum_{=0}^{n} \frac{1}{3}(A + A_{+1} + \sqrt{A \times A_{+1}}) \times \triangle L$$

- $V$
- $A$
- $\triangle L$ +

$d$

$$
\begin{aligned}
CA(X) &= \frac{1}{3}(\frac{1}{2}d \times h + d \times h) \times \frac{h(X)}{k(X)} \\
&= \frac{h(X)^2}{3k(X)}(\frac{3}{2}d) \\
&= \frac{h(X)^2 d}{2k(X)} \qquad ( )
\end{aligned}
$$

**building cost**

$P = \rho g h$

**dam**, **reser-voir**

$\beta$,

$\sigma$

$$Cost(X) = \lambda \frac{\rho g h(X) \times d \times h(X)}{2\sigma} + \beta$$
$$= \lambda \frac{\rho g d h(X)^2}{2\sigma} + \beta \qquad ( )$$

$\lambda$ fi

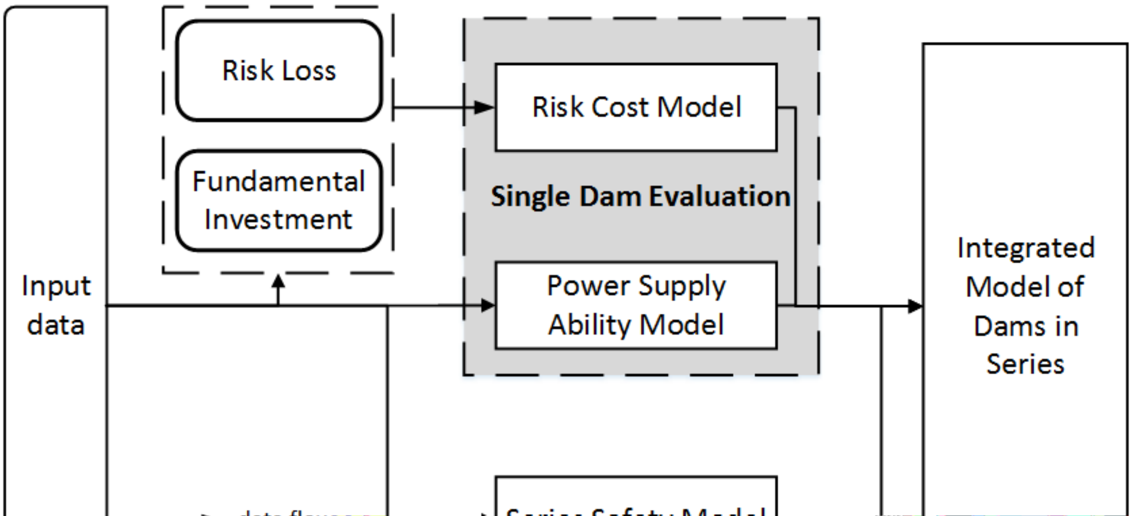
## 4.3 iMoDS: Integrated Model of Dams in Series

fi

### 4.3.1 Prerequisites

$Q(X;t)$

$Q(X;t)$ fl $t$ $i$. $Q(X;t)$ $Q(X;\bar{t})$, fl $d$ fl fi fl




[redacted]

$k(X) = \frac{dE \quad n( )}{d}|$ $i$

, $X$. $i$
$k(X)$.

$k(X)$

### 4.3.2 Risk Cost Model

**cost** **safety**. fi

*the extent that a person focus more on safety than merely cost.*

**Cost - Fundamental Investment**

, $P$

$$= \frac{Cost(X)}{P^{1.1}} \quad ( )$$

$P$ ,

**Safety - Risk Loss** **Risk Loss**. $[1-(1-P) \ ]$ , fi $i$

**collapse probability * (1+ indirect damage index)*Volumetric flow rate**

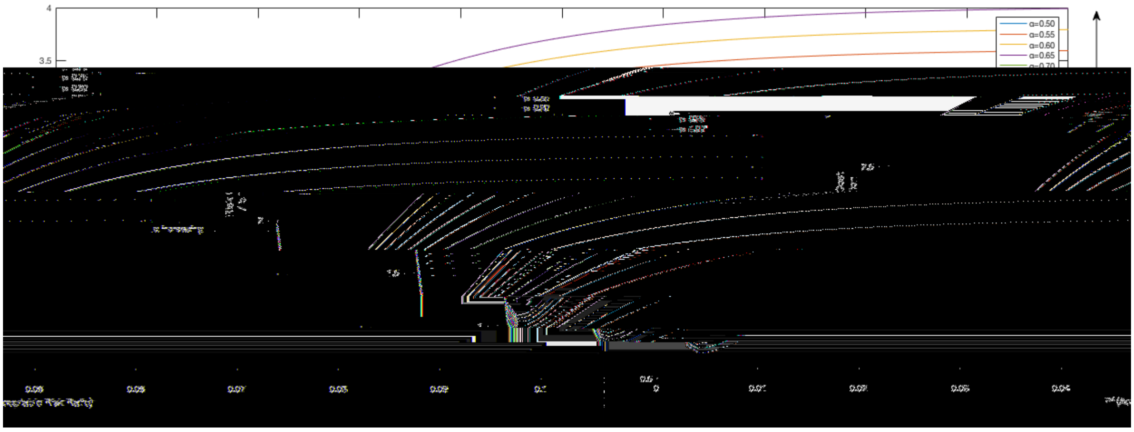
$$= [1-(1-P) \ ]\alpha(1+\lambda)Q(X;\bar{t}) \quad ( )$$

$\alpha$ fi

$$Risk(X) = \quad +$$

$P$ **Risk Value**.

$\alpha$. , $\alpha$




$X$ $P$ **Risk Value** $\alpha$

**Observation 1**

**Illustraion** **Risk Value** **Risk Loss** $P$ **Fundamental Investment** $P$.

**Observation 2** $\alpha$ **Risk Cost**

**Illustraion** fi $\alpha$ $P$ $P$

### 4.3.3 Series Safety Model

fi

$$Series(X) = \frac{\overline{d}}{e^{\sum_{i=1}^{n-1} \frac{(d_i - \overline{d})^2}{\overline{d}^2}}} \quad ( )$$

$\overline{d}$

#### 4.3.4 Power Supply Ability Model

$$Supply(X) = \eta \rho g h(X) Q(X; \bar{t}) \qquad ( )$$

- $\eta$
- $\rho$
- $g$

#### 4.3.5 Integration

- $Series(X)$ **Positively** $Series(X)'$
- $Risk(X)$ $Risk(X)'$
- $Supply(X)$ **Positively** $Supply(X)'$

$[0, 1]$.

$$x' = \frac{x - \min(x)}{\max(x) - \min(x)}$$

- $x$
- $x'$

, $Series(X\ )$, $Risk(X\ )$, $Supply(X\ )$ $Series(X\ )'$, $Risk(X\ )'$, $Supply(X\ )'$ , $Risk(X\ )$ **negatively** **positively** fi

$$Series(X)' = Series(X) \times \frac{n-1}{L}$$
$$Risk(X\ )' = \min(Risk(X\ ))/Risk(X\ )$$
$$Supply(X)' = Supply(X) \times \frac{n-1}{L}$$

**Final Value** , $Supply(X\ )$ $Series(X)$ $[-1, 0]$.

**Integrated Model of Dams in Series**

$$\begin{aligned} \max \quad & = Series(X)' + \sum_{=1}^{n} Risk(X\ )' + \sum_{=1}^{n} Supply(X\ )' \\ \text{s.t} \quad & X\ \le L, \qquad \forall X\ \in [1, n] \\ & \sum_{=1}^{n} CA(X\ ) \ge M \\ & 10 \le n \le 20, \qquad n \in \mathbb{Z} \\ & X\ \ge 0, \qquad \forall X\ \in [1, n] \end{aligned}$$

$$\begin{aligned} & = Series(X)' + \sum_{=1}^{n} Risk(X\ )' + \sum_{=1}^{n} Supply(X\ )' \\ & = \frac{\bar{d}}{e^{\sum_{i=1}^{n-1} \frac{(d_i - \bar{d})^2}{\bar{d}^2}}} \times \frac{n-1}{L} + \sum_{=1}^{n} \min(Risk(X\ ))/\{\frac{Cost(X\ )}{P^{1.1}} + \\ & [1 - (1 - P\ )\ \ ]\alpha(1+\lambda)Q(X\ ; \bar{t})\} + \sum_{=1}^{n} \rho g \eta L k(X\ ) Q(X\ ; \bar{t}) \end{aligned}$$

#### 4.3.6 Ranking Submodels with AHP

( )

| |
|---|
| |

$$\max \quad = 0.35Series(X)' + 0.54\sum_{=1}^{n} Risk(X\ )' + 0.11\sum_{=1}^{n} Supply(X\ )'$$

$$\text{s.t} \quad X\ \leq L, \qquad \forall X\ \in [1, n]$$

$$\sum_{=1}^{n} CA(X\ ) \geq M$$

$$10 \leq n \leq 20, \qquad n \in \mathbb{Z}$$

$$X\ \geq 0, \qquad \forall X\ \in [1, n]$$

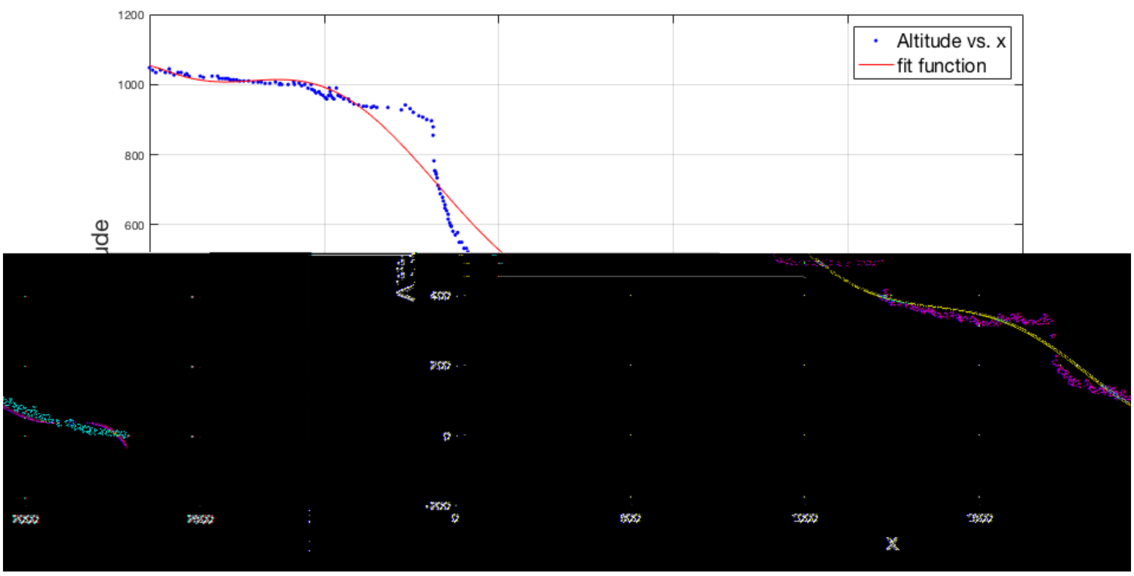
# 5 Implementation

## 5.1 Data

fi fi fi fi fi

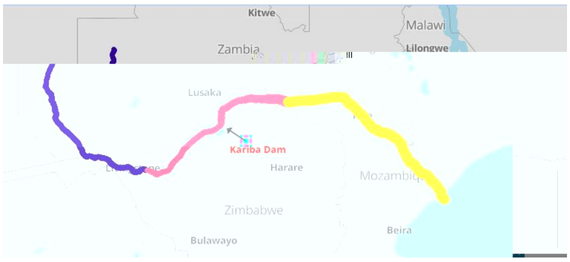
**eighth-power polynomials**.

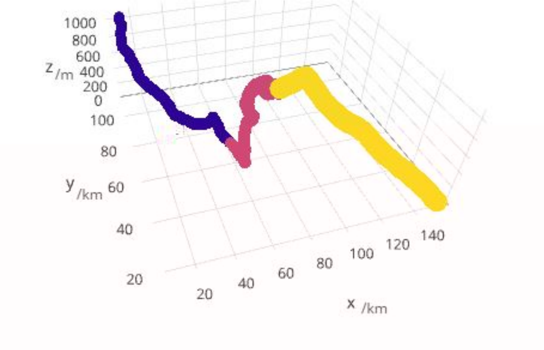
$f(X\ )$




fl fl

$Q(X\ ;\bar{t})$




( ) -( )

## 5.2 Number and placement of the new dams

### 5.2.1 Whole Procedures

$\{X\}$

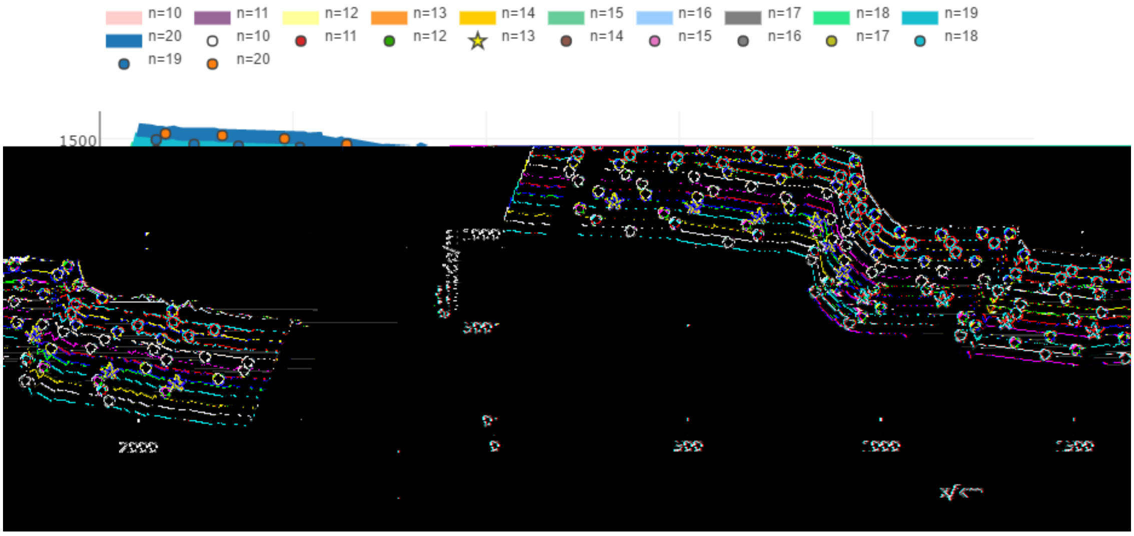
$\{Xi\}$

### 5.2.2 Determining the number and placement by genetic algorithm

$h(X)$, $\{Xi\}$, $P$, $n$ **genetic algorithm**

**Risk Cost**

$M$, $P$




**Power Supply**,

, **Series**

fi

$n = 13$ $P = 0.0024$,

$n$

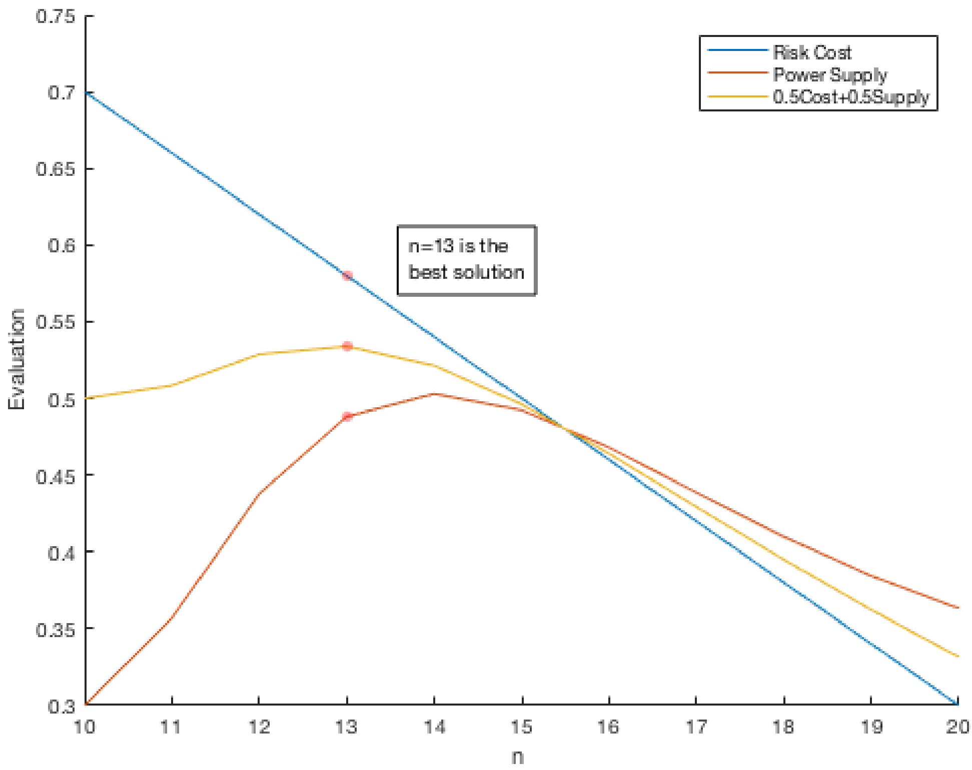
$\sum \frac{\min( \quad (\ _i))}{( \ _i)}$ **decreases**.

$\beta$

$\sum Supply(X\ )$ **increases and then decreases**. $n$

$n$ , $Series(X)$

$n$




**Final Value** $n$

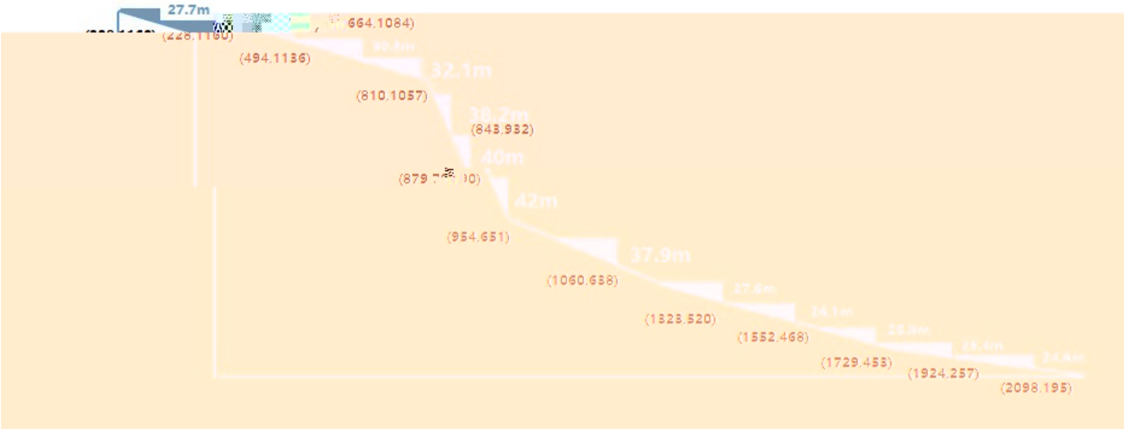
$Series(X)$ $n$.

fi $n = 13$ $P_f = 0.0024$.

1 13, $X$ $i$

### 5.2.3 Determining the height of each dam

fi fl **risk cost** **power supply**, **final value** **(iMoDS model)** $P$, $h(X)$. $Series(X)$.




( )

$$Supply(X\ )' - Risk(X\ )' = 0$$

$$\eta\rho g Q(X\ ;\bar{t}) - \lambda \frac{2\rho g d h(X\ )}{2\sigma P^{1.1}} = 0$$

$$h(X\ ) = \frac{\eta\sigma P^{1.1}}{\lambda d} Q(X\ ;\bar{t}) \qquad (\ )$$

fi $Q(X\ ;\bar{t})$ $h(X\ )$ $h(X\ )$.

$$\sum_{=1}^{n} CA(X\ ) = \sum_{=1}^{n} \frac{h(X\ )^2 d}{2k(X\ )} \geq M \qquad (\ )$$

. **If** $\{X\ \}$ fi

. **Else,** $X$

**Thus, Final Value of iMoDS model and each dam's height are both determined by** $\{X\ \}$.

fi

# 6 Strategies

## 6.1 A Balance Between Safety and Costs

, **Risk Cost, Power Supply,** **Series Safety**. **Risk Cost** $P$ . **Series Safety** fi

## 6.2 Protection for Lake Kariba

fi fi

$$Protection = \sum_{=1}^{n} CA(X\ ) N\ (X_L\quad , \sigma^2) \quad (\ \ )$$

$X_L$ , $\sigma$ fi

**Protection**

## 6.3 Guidance for Emergency Water Flow Situations

fl ( . . fl ) fi fi

fl fl fl -

**Method** fi fi

fl fl fi fi fl

**Method**

fi

## 6.4 Guidance for Extreme Water Flows

$Q(X;\bar{t})$ fl $Q(X;t)$ $t$.

$T$. $p, k,$ $m$. $p$ $k$ $m$ fi

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -p - km$$

$$m = \frac{ce^{-} - p}{k}$$

$$m(0) = M, m(T) = 0$$

$$p = \frac{Mke^{-}}{1 - e^{-}}$$

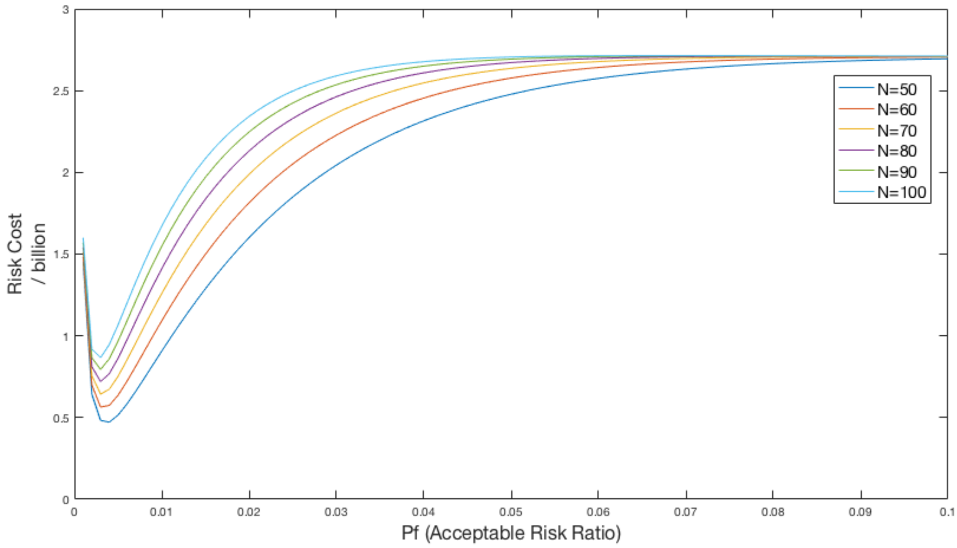
$p$.

fi fi $Q(X;t)$

# 7 Model Analysis

## 7.1 Sensitivity Analysis

### 7.1.1 Impact of Planned Working Years $N$ for a Dam




$X$ $P$ **Risk Value** $N$

$N$ fi

**Risk Loss**

$N$.

### 7.1.2 Impact of Extreme Condition ratio $\alpha$

$\alpha$.

## 7.2 Strengths and Weaknesses

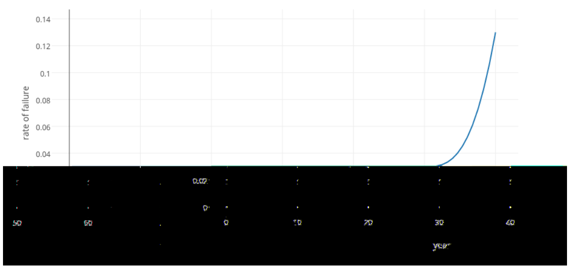
### 7.2.1 Strengths

### 7.2.2 Weaknesses

# Brief Assessment Report

$3 \times 10^{-4}$.

$$FR(T) = \begin{cases} 3 \times 10^{-4} & 0 \le T < 50 \\ 1 - e^{-(\frac{t+143.67}{100})^3} & T \ge 50 \end{cases} \qquad (\ )$$




## Option 1: Repairing the existing Kariba Dam

**Potential Cost** $

$$FR(t) = 1 - e^{-(\frac{t+143.67}{100})^3}, \qquad (\ )$$

$t =$

$$294 + C \times \int_0^{-2017} FR(t)\mathrm{d}t \qquad (\ )$$

**Benefits** fi

**Shortcomings**

### Option 2: Rebuilding the existing Kariba Dam

**Potential Cost** $ ( ) 27.2 241.4 $241.4/27.2 \times \$1.8 = \$15.975$

fi $Y$ $C$,

$$7440 + C \times \int_0^{-2017} FR(t)\mathrm{d}t \qquad (\ )$$

**Benefits**

**Shortcomings**

### Option 3: Removing the Kariba Dam and replacing it with a series of ten to twenty smaller dams along the Zambezi River

**Potential Costs**

$= * +$

$n$ $n$ $10\%$ $15\%$ $\$15.975 * (1 + 10\% * n + 15\%) = \$34.363$ $\$50.321$.

$n = 13$, $\$39.139$.

**Benefits** fl fl

**Shortcomings**

# References

*Engineering Mechanics(In Chinese)*, ( ),

*Journal of Water Resources Planning and Management*, ( ),

*Journal of Water Resources Planning and Management*, ( ),

, *Principles of macroeconomics*.

# Appendices

## Appendix A Implemented Genetic Algorithm

```
n=13;% need adjust
B_sub1=[zeros(n,1);0.00001];
%B_sub1 means the VLB of the Xi
B_sub2=[ones(n,1)*2100;0.1];
%B_sub2 means the VUB of the Xi
B=[B_sub1,B_sub2];
% constraint of Xi
initPop=initializega(n,B,'fitness');
[x endPop,bPop,trace]=ga(B,'fitness',[],initPop,[1e-6 1 1],
    'maxGenTerm',10000,'normGeomSelect',...
    [0.08],['arithXover'],[2],'nonUnifMutation',[2 10000 3]);
%3000 generations
xx=sort(x(1:14));
% sites selection sorts as ascending order
x=[xx,x(15)];
% the last one is the evaluation value
```

## Appendix B Fitness Function

```
function[sol,eval]=fitness(sol,~)
n=13;
x=sol(1,1:n+1);
xx=sort(x(1:n));
x=[xx,x(n+1)];
p1=0;
p2=0;
cc=0;
N=50;
Ca=300;
BB=6000;
aa1=400;
rr=0.8;
pp=1005;
yy=0.5;
l=200;
Cmin=2e6;
Pm=1e8;
% above are the constant parameter setting
wei=[0.12,0.54,0.34];
% wei means the weight of three key function
for i=1:n
    k=my(x(i));
    Q=my2(x(i));
    R=(1-(1-x(n+1))^N)*Q*aa1*(1+rr);
    C=((Ca/n)*k+BB)*(1/(x(n+1))^1.1);
    Power=Q*pp*yy*k*l*9.8/1000;
    p1=p1+Power;
    p2=p2+C+R;
end
p1=p1/Pm
p2=Cmin/p2
```

```
for j=1:n-1
    cc=cc+((x(n)-x(1))/(n-1)-(x(j+1)-x(j)))^2;
end
    Sa=((x(n)-x(1))/exp(cc/((x(n)-x(1))/(n-1))^2))/2100
    eval=p1*wei(1)+Sa*wei(3)+p2*wei(2)
end
```